```
clear      % clears all variables from the workspace.
basemva = 100;  accuracy = 0.001;  maxiter = 20;

busdata =  [ same as in Example 6.9 ];
linedata = [ same as in Example 6.9 ];

     lfybus                  % Forms the bus admittance matrix
     decouple   % Power flow solution by fast decoupled method
     busout     % Prints the power flow solution on the screen
     lineflow % Computes and displays the line flow and losses
```

The output of **decouple** is

```
            Power Flow Solution by Fast Decoupled Method
               Maximum Power mismatch = 0.000919582
                      No. of iterations = 15

Bus Voltage  Angle    -----Load-----   --Generation-- Injected
No. Mag.     Degree     MW      Mvar     MW      Mvar     Mvar

 1   1.060    0.000    0.000    0.000  260.998  -17.021   0.00
 2   1.043   -5.497   21.700   12.700   40.000   48.822   0.00
 3   1.022   -8.004    2.400    1.200    0.000    0.000   0.00
 4   1.013   -9.662    7.600    1.600    0.000    0.000   0.00
 5   1.010  -14.381   94.200   19.000    0.000   35.975   0.00
 6   1.012  -11.398    0.000    0.000    0.000    0.000   0.00
 7   1.003  -13.149   22.800   10.900    0.000    0.000   0.00
 8   1.010  -12.115   30.000   30.000    0.000   30.828   0.00
 9   1.051  -14.434    0.000    0.000    0.000    0.000   0.00
10   1.044  -16.024    5.800    2.000    0.000    0.000  19.00
11   1.082  -14.434    0.000    0.000    0.000   16.120   0.00
12   1.057  -15.303   11.200    7.500    0.000    0.000   0.00
13   1.071  -15.303    0.000    0.000    0.000   10.421   0.00
14   1.042  -16.198    6.200    1.600    0.000    0.000   0.00
15   1.038  -16.276    8.200    2.500    0.000    0.000   0.00
16   1.045  -15.881    3.500    1.800    0.000    0.000   0.00
17   1.039  -16.188    9.000    5.800    0.000    0.000   0.00
18   1.028  -16.882    3.200    0.900    0.000    0.000   0.00
19   1.025  -17.051    9.500    3.400    0.000    0.000   0.00
20   1.029  -16.852    2.200    0.700    0.000    0.000   0.00
21   1.032  -16.468   17.500   11.200    0.000    0.000   0.00
22   1.033  -16.454    0.000    0.000    0.000    0.000   0.00
23   1.027  -16.661    3.200    1.600    0.000    0.000   0.00
24   1.022  -16.829    8.700    6.700    0.000    0.000   4.30
25   1.019  -16.423    0.000    0.000    0.000    0.000   0.00
26   1.001  -16.840    3.500    2.300    0.000    0.000   0.00
```

```
27   1.026  -15.912    0.000    0.000    0.000    0.000   0.00
28   1.011  -12.057    0.000    0.000    0.000    0.000   0.00
29   1.006  -17.136    2.400    0.900    0.000    0.000   0.00
30   0.995  -18.014   10.600    1.900    0.000    0.000   0.00

Total                283.400  126.200  300.998  125.145  23.30
```

The output of the **lineflow** is the same as the line flow output of Example 6.9 with the power mismatch as dictated by the fast decoupled method.

## PROBLEMS

**6.1.** A power system network is shown in Figure 6.17. The generators at buses 1 and 2 are represented by their equivalent current sources with their reactances in per unit on a 100-MVA base. The lines are represented by $\pi$ model where series reactances and shunt reactances are also expressed in per unit on a 100 MVA base. The loads at buses 3 and 4 are expressed in MW and Mvar.

(a) Assuming a voltage magnitude of 1.0 per unit at buses 3 and 4, convert the loads to per unit impedances. Convert network impedances to admittances and obtain the bus admittance matrix by inspection.

(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.)

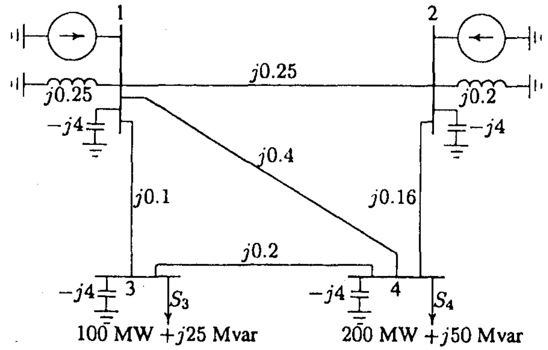

**FIGURE 6.17**
One-line diagram for Problem 6.1.

**6.2.** A power system network is shown in Figure 6.18. The values marked are impedances in per unit on a base of 100 MVA. The currents entering buses 1 and 2 are

$$I_1 = 1.38 - j2.72 \text{ pu}$$
$$I_2 = 0.69 - j1.36 \text{ pu}$$

(a) Determine the bus admittance matrix by inspection.
(b) Use the function **Y = ybus(zdata)** to obtain the bus admittance matrix. The function argument **zdata** is a matrix containing the line bus numbers, resistance and reactance. (See Example 6.1.) Write the necessary *MATLAB* commands to obtain the bus voltages.

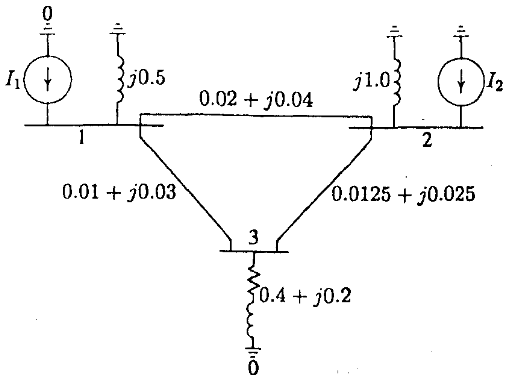

**FIGURE 6.18**
One-line diagram for Problem 6.2.

**6.3.** Use Gauss-Seidel method to find the solution of the following equations

$$x_1 + x_1 x_2 = 10$$
$$x_1 + x_2 = 6$$

with the following initial estimates
(a) $x_1^{(0)} = 1$ and $x_2^{(0)} = 1$
(b) $x_1^{(0)} = 1$ and $x_2^{(0)} = 2$
Continue the iterations until $|\Delta x_1^{(k)}|$ and $|\Delta x_2^{(k)}|$ are less than 0.001.

**6.4.** A fourth-order polynomial equation is given by

$$x^4 - 21x^3 + 147x^2 - 379x + 252 = 0$$

(a) Use Newton-Raphson method and hand calculations to find one of the roots of the polynomial equation. Start with the initial estimate of $x^{(0)} = 0$ and continue until $|\Delta x^{(k)}| < 0.001$.
(b) Write a *MATLAB* program to find the roots of the above polynomial by Newton-Raphson method. The program should prompt the user to input the initial estimate. Run using the initial estimates of 0, 3, 6, 10.
(c) Check your answers using the *MATLAB* function **r = roots(A)**, where **A** is a row vector containing the polynomial coefficients in descending powers.

**6.5.** Use Newton-Raphson method and hand calculation to find the solution of the following equations:

$$x_1^2 - 2x_1 - x_2 = 3$$
$$x_1^2 + x_2^2 = 41$$

(a) Start with the initial estimates of $x_1^{(0)} = 2$, $x_2^{(0)} = 3$. Perform three iterations.
(b) Write a *MATLAB* program to find one of the solutions of the above equations by Newton-Raphson method. The program should prompt the user to input the initial estimates. Run the program with the above initial estimates.

**6.6.** In the power system network shown in Figure 6.19, bus 1 is a slack bus with $V_1 = 1.0\angle 0°$ per unit and bus 2 is a load bus with $S_2 = 280$ MW + $j60$ Mvar. The line impedance on a base of 100 MVA is $Z = 0.02 + j0.04$ per unit.
(a) Using Gauss-Seidel method, determine $V_2$ . Use an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and perform four iterations.
(b) If after several iterations voltage at bus 2 converges to $V_2 = 0.90 - j0.10$, determine $S_1$ and the real and reactive power loss in the line.

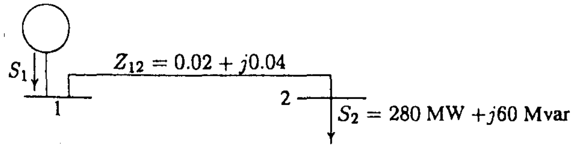

**FIGURE 6.19**
One-line diagram for Problem 6.6.

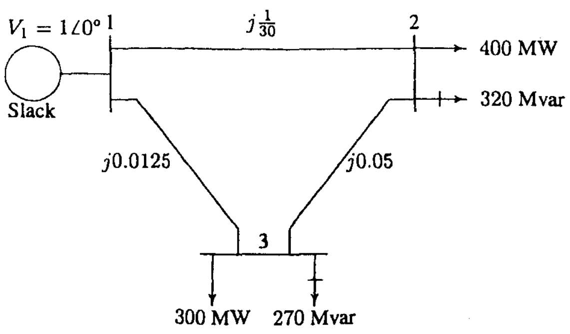

**FIGURE 6.20**
One-line diagram for Problem 6.7.

**6.7.** Figure 6.20 shows the one-line diagram of a simple three-bus power system with generation at bus 1. The voltage at bus 1 is $V_1 = 1.0\angle 0°$ per unit. The scheduled loads on buses 2 and 3 are marked on the diagram. Line impedances are marked in per unit on a 100-MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, determine $V_2$ and $V_3$. Perform two iterations.

(b) If after several iterations the bus voltages converge to

$$V_2 = 0.90 - j0.10 \ \text{pu}$$
$$V_3 = 0.95 - j0.05 \ \text{pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.

(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.) Use a power accuracy of 0.00001 and an acceleration factor of 1.0.

**6.8.** Figure 6.21 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 3. The voltage at bus 1 is $V_1 = 1.025\angle 0°$ per unit. Voltage magnitude at bus 3 is fixed at 1.03 pu with a real power generation of 300 MW. A load consisting of 400 MW and 200 Mvar is taken from bus 2. Line impedances are marked in per unit on a 100-MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

(a) Using Gauss-Seidel method and initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.03 + j0$ and keeping $|V_3| = 1.03$ pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.

(b) If after several iterations the bus voltages converge to

$$V_2 = 1.001243\angle -2.1° = 1.000571 - j0.0366898 \ \text{pu}$$
$$V_3 = 1.03\angle 1.36851° = 1.029706 + j0.0246 \ \text{pu}$$

determine the line flows and line losses and the slack bus real and reactive power. Construct a power flow diagram and show the direction of the line flows.

(c) Check the power flow solution using the **lfgauss** and other required programs. (Refer to Example 6.9.)

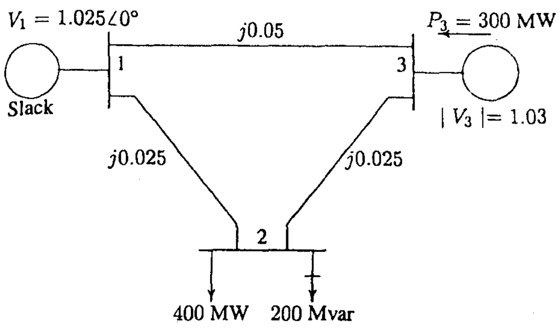

**FIGURE 6.21**
One-line diagram for Problem 6.8.

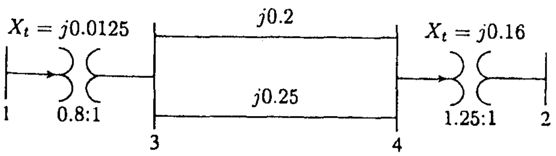

**FIGURE 6.22**
One-line diagram for Problem 6.9.

**6.9.** The one-line diagram of a four-bus power system is as shown in Figure 6.22. Reactances are given in per unit on a common MVA base. Transformers $T_1$ and $T_2$ have tap settings of 0.8:1, and 1.25:1 respectively. Obtain the bus admittance matrix.

**6.10.** In the two-bus system shown in Figure 6.23, bus 1 is a slack bus with $V_1 = 1.0\angle 0^\circ$ pu. A load of 150 MW and 50 Mvar is taken from bus 2. The line admittance is $y_{12} = 10\angle -73.74^\circ$ pu on a base of 100 MVA. The expression for real and reactive power at bus 2 is given by

$$P_2 = 10|V_2||V_1|\cos(106.26^\circ - \delta_2 + \delta_1) + 10|V_2|^2\cos(-73.74^\circ)$$
$$Q_2 = -10|V_2||V_1|\sin(106.26^\circ - \delta_2 + \delta_1) - 10|V_2|^2\sin(-73.74^\circ)$$

Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0^\circ$. Perform two iterations.

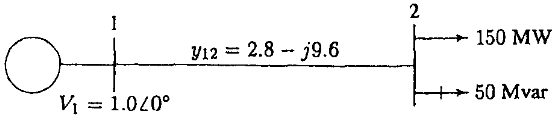

**FIGURE 6.23**
One-line diagram for Problem 6.10.

**6.11.** In the two-bus system shown in Figure 6.24, bus 1 is a slack bus with $V_1 = 1.0\angle 0^\circ$ pu. A load of 100 MW and 50 Mvar is taken from bus 2. The line impedance is $z_{12} = 0.12 + j0.16$ pu on a base of 100 MVA. Using Newton-Raphson method, obtain the voltage magnitude and phase angle of bus 2. Start with an initial estimate of $|V_2|^{(0)} = 1.0$ pu and $\delta_2^{(0)} = 0^\circ$. Perform two iterations.

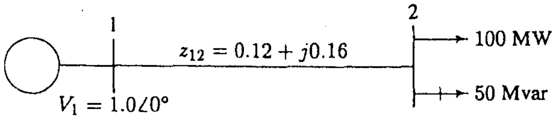

**FIGURE 6.24**
One-line diagram for Problem 6.11.

**6.12.** Figure 6.25 shows the one-line diagram of a simple three-bus power system with generation at buses 1 and 2. The voltage at bus 1 is $V = 1.0\angle 0^\circ$ per unit. Voltage magnitude at bus 2 is fixed at 1.05 pu with a real power generation of 400 MW. A load consisting of 500 MW and 400 Mvar is taken from bus 3. Line admittances are marked in per unit on a 100 MVA base. For the purpose of hand calculations, line resistances and line charging susceptances are neglected.

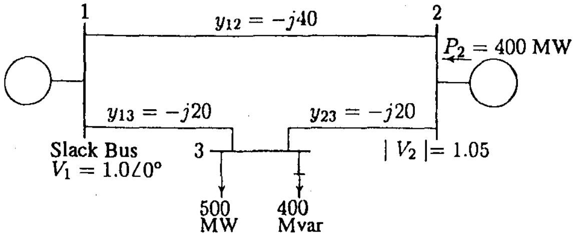

**FIGURE 6.25**
One-line diagram for Problem 6.12

(a) Show that the expression for the real power at bus 2 and real and reactive power at bus 3 are

$$P_2 = 40|V_2||V_1|\cos(90^\circ - \delta_2 + \delta_1) + 20|V_2||V_3|\cos(90^\circ - \delta_2 + \delta_3)$$
$$P_3 = 20|V_3||V_1|\cos(90^\circ - \delta_3 + \delta_1) + 20|V_3||V_2|\cos(90^\circ - \delta_3 + \delta_2)$$
$$Q_3 = -20|V_3||V_1|\sin(90^\circ - \delta_3 + \delta_1) - 20|V_3||V_2|\sin(90^\circ - \delta_3 + \delta_2) + 40|V_3|^2$$

(b) Using Newton-Raphson method, start with the initial estimates of $V_2^{(0)} = 1.0 + j0$ and $V_3^{(0)} = 1.0 + j0$, and keeping $|V_2| = 1.05$ pu, determine the phasor values of $V_2$ and $V_3$. Perform two iterations.

(c) Check the power flow solution for Problem 6.12 using **lfnewton** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

**6.13.** For Problem 6.12:

(a) Obtain the power flow solution using the fast decoupled algorithm. Perform two iterations.

(b) Check the power flow solution for Problem 6.12 using **decouple** and other required programs. Assume the regulated bus (bus # 2) reactive power limits are between 0 and 600 Mvar.

**6.14.** The 26-bus power system network of an electric utility company is shown in Figure 6.26 (page 256). Obtain the power flow solution by the following methods:

(a) Gauss-Seidel power flow (see Example 6.9).
(b) Newton-Raphson power flow (see Example 6.11).
(c) Fast decoupled power flow (see Example 6.13).

The load data is as follows.

| LOAD DATA | | | | | |
|---|---|---|---|---|---|
| Bus | Load | | Bus | Load | |
| No. | MW | Mvar | No. | MW | Mvar |
| 1 | 51.0 | 41.0 | 14 | 24.0 | 12.0 |
| 2 | 22.0 | 15.0 | 15 | 70.0 | 31.0 |
| 3 | 64.0 | 50.0 | 16 | 55.0 | 27.0 |
| 4 | 25.0 | 10.0 | 17 | 78.0 | 38.0 |
| 5 | 50.0 | 30.0 | 18 | 153.0 | 67.0 |
| 6 | 76.0 | 29.0 | 19 | 75.0 | 15.0 |
| 7 | 0.0 | 0.0 | 20 | 48.0 | 27.0 |
| 8 | 0.0 | 0.0 | 21 | 46.0 | 23.0 |
| 9 | 89.0 | 50.0 | 22 | 45.0 | 22.0 |
| 10 | 0.0 | 0.0 | 23 | 25.0 | 12.0 |
| 11 | 25.0 | 15.0 | 24 | 54.0 | 27.0 |
| 12 | 89.0 | 48.0 | 25 | 28.0 | 13.0 |
| 13 | 31.0 | 15.0 | 26 | 40.0 | 20.0 |

Voltage magnitude, generation schedule, and the reactive power limits for the regulated buses are tabulated below. Bus 1, whose voltage is specified as $V_1 = 1.025\angle 0^\circ$, is taken as the slack bus.

| GENERATION DATA | | | | |
|---|---|---|---|---|
| Bus | Voltage | Generation | Mvar Limits | |
| No. | Mag. | MW | Min. | Max. |
| 1 | 1.025 | | | |
| 2 | 1.020 | 79.0 | 40.0 | 250.0 |
| 3 | 1.025 | 20.0 | 40.0 | 150.0 |
| 4 | 1.050 | 100.0 | 40.0 | 80.0 |
| 5 | 1.045 | 300.0 | 40.0 | 160.0 |
| 26 | 1.015 | 60.0 | 15.0 | 50.0 |

The Mvar of the shunt capacitors installed at substations and the transformer tap settings are given below.

| SHUNT CAPACITORS | |
|---|---|
| Bus No. | Mvar |
| 1 | 4.0 |
| 4 | 2.0 |
| 5 | 5.0 |
| 6 | 2.0 |
| 11 | 1.5 |
| 12 | 2.0 |
| 15 | 0.5 |
| 19 | 5.0 |

| TRANSFORMER TAP | |
|---|---|
| Designation | Tap Setting |
| 2 – 3 | 0.960 |
| 2 – 13 | 0.960 |
| 3 – 13 | 1.017 |
| 4 – 8 | 1.050 |
| 4 – 12 | 1.050 |
| 6 – 19 | 0.950 |
| 7 – 9 | 0.950 |

The line and transformer data containing the series resistance and reactance in per unit and one-half the total capacitance in per unit susceptance on a 100-MVA base are tabulated below.

| LINE AND TRANSFORMER DATA | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Bus No. | Bus No. | $R$, pu | $X$, pu | $\frac{1}{2}B$, pu | Bus No. | Bus No. | $R$, pu | $X$, pu | $\frac{1}{2}B$, pu |
| 1 | 2 | 0.0005 | 0.0048 | 0.0300 | 10 | 22 | 0.0069 | 0.0298 | 0.005 |
| 1 | 18 | 0.0013 | 0.0110 | 0.0600 | 11 | 25 | 0.0960 | 0.2700 | 0.010 |
| 2 | 3 | 0.0014 | 0.0513 | 0.0500 | 11 | 26 | 0.0165 | 0.0970 | 0.004 |
| 2 | 7 | 0.0103 | 0.0586 | 0.0180 | 12 | 14 | 0.0327 | 0.0802 | 0.000 |
| 2 | 8 | 0.0074 | 0.0321 | 0.0390 | 12 | 15 | 0.0180 | 0.0598 | 0.000 |
| 2 | 13 | 0.0035 | 0.0967 | 0.0250 | 13 | 14 | 0.0046 | 0.0271 | 0.001 |
| 2 | 26 | 0.0323 | 0.1967 | 0.0000 | 13 | 15 | 0.0116 | 0.0610 | 0.000 |
| 3 | 13 | 0.0007 | 0.0054 | 0.0005 | 13 | 16 | 0.0179 | 0.0888 | 0.001 |
| 4 | 8 | 0.0008 | 0.0240 | 0.0001 | 14 | 15 | 0.0069 | 0.0382 | 0.000 |
| 4 | 12 | 0.0016 | 0.0207 | 0.0150 | 15 | 16 | 0.0209 | 0.0512 | 0.000 |
| 5 | 6 | 0.0069 | 0.0300 | 0.0990 | 16 | 17 | 0.0990 | 0.0600 | 0.000 |
| 6 | 7 | 0.0053 | 0.0306 | 0.0010 | 16 | 20 | 0.0239 | 0.0585 | 0.000 |
| 6 | 11 | 0.0097 | 0.0570 | 0.0001 | 17 | 18 | 0.0032 | 0.0600 | 0.038 |
| 6 | 18 | 0.0037 | 0.0222 | 0.0012 | 17 | 21 | 0.2290 | 0.4450 | 0.000 |
| 6 | 19 | 0.0035 | 0.0660 | 0.0450 | 19 | 23 | 0.0300 | 0.1310 | 0.000 |
| 6 | 21 | 0.0050 | 0.0900 | 0.0226 | 19 | 24 | 0.0300 | 0.1250 | 0.002 |
| 7 | 8 | 0.0012 | 0.0069 | 0.0001 | 19 | 25 | 0.1190 | 0.2249 | 0.004 |
| 7 | 9 | 0.0009 | 0.0429 | 0.0250 | 20 | 21 | 0.0657 | 0.1570 | 0.000 |
| 8 | 12 | 0.0020 | 0.0180 | 0.0200 | 20 | 22 | 0.0150 | 0.0366 | 0.000 |
| 9 | 10 | 0.0010 | 0.0493 | 0.0010 | 21 | 24 | 0.0476 | 0.1510 | 0.000 |
| 10 | 12 | 0.0024 | 0.0132 | 0.0100 | 22 | 23 | 0.0290 | 0.0990 | 0.000 |
| 10 | 19 | 0.0547 | 0.2360 | 0.0000 | 22 | 24 | 0.0310 | 0.0880 | 0.000 |
| 10 | 20 | 0.0066 | 0.0160 | 0.0010 | 23 | 25 | 0.0987 | 0.1168 | 0.000 |

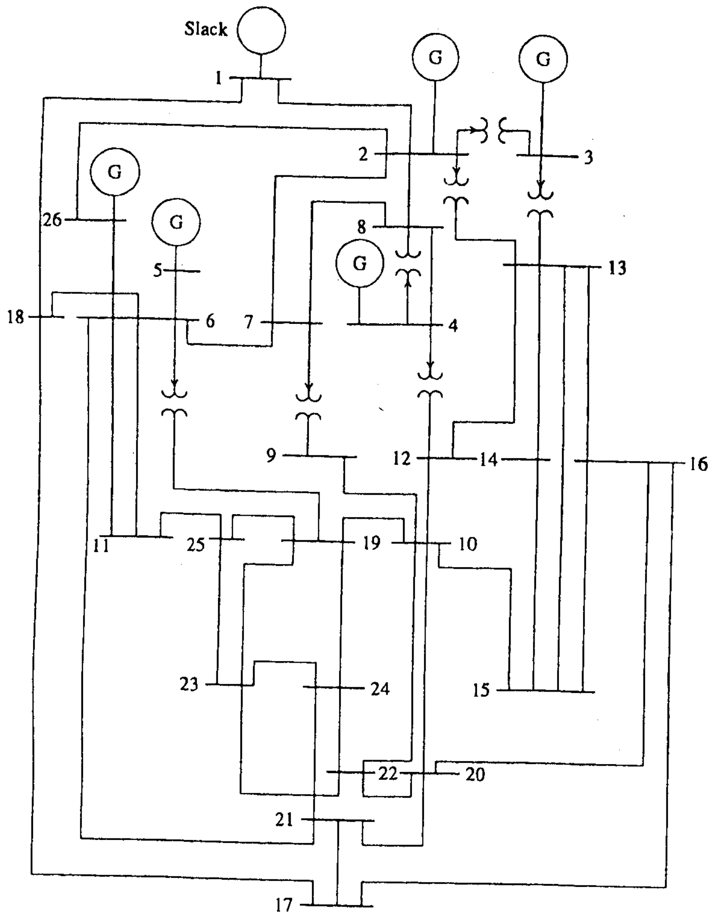

**FIGURE 6.26**
One-line diagram for Problem 6.14.

# CHAPTER 7

# OPTIMAL DISPATCH OF GENERATION

## 7.1 INTRODUCTION

The formulation of power flow problem and its solutions were discussed in Chapter 6. One type of bus in the power flow was the voltage-controlled bus, where real power generation and voltage magnitude were specified. The power flow solution provided the voltage phase angle and the reactive power generation. In a practical power system, the power plants are not located at the same distance from the center of loads and their fuel costs are different. Also, under normal operating conditions, the generation capacity is more than the total load demand and losses. Thus, there are many options for scheduling generation. In an interconnected power system, the objective is to find the real and reactive power scheduling of each power plant in such a way as to minimize the operating cost. This means that the generator's real and reactive power are allowed to vary within certain limits so as to meet a particular load demand with minimum fuel cost. This is called the *optimal power flow* (OPF) problem. The OPF is used to optimize the power flow solution of large scale power system. This is done by minimizing selected objective functions while maintaining an acceptable system performance in terms of generator capability limits and the output of the compensating devices. The objective functions, also