Steady State Analysis of an induction generator infinite bus system

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***Abstract*— This paper conducts a fundamental analysis on the induction generator infinite bus system which is a useful representation for a wind energy converter interfaced to a utility through a transmission line. A third order dynamic model is used to represent the induction generator and the resultant nonlinear system equations are analyzed to derive a condition that guarantees the existence of equilibrium points (or steady state solutions) to the dynamic system. This condition is used to derive three other auxiliary conditions which compute (i) the minimum value of capacitance, (ii) the maximum deliverable power and (iii) the maximum external reactance that can be connected to the machine. It is also shown that terminal voltage regulation has a strong restrictive influence on each the issues above. The analysis presented could be a useful tool for preliminary planning studies involving wind energy converters.**

Index terms : Induction generators, wind energy conversion

NOMENCLATURE

- r_s : armature (stator) resistance
- x_1 : armature leakage reactance
- x_m : magnetizing reactance
- x_2 : rotor leakage reactance
- r_r : rotor resistance
- ω_s : angular speed of the synchronously rotating frame
- s : slip of the machine
- E' : equivalent voltage source of the machine
- I_s : armature current the machine
- U : terminal voltage of the machine
- T_o' : rotor open circuit time constant
- x' : transient reactance
- P_e : electrical power output of the machine
- P_m : input mechanical power to the machine
- H : inertia constant
- I_c : current through the compensator bank
- E_b : magnitude of the infinite bus voltage
- r_e : resistance of the transmission line
- x_e : reactance of the transmission line
- Y_c: admittance of the capacitor bank

I. INTRODUCTION

WIND energy conversion has emerged as a viable alternative to meet the increased demand for energy resources in the recent years. There are different configurations for wind turbine units which include synchronous, asynchronous generators, pitch regulated, or stall regulated systems. A standalone scheme for wind energy conversion employing batteries for energy storage and a permanent magnet synchronous generator was studied in [1] where the wind turbine was the only source of power. However, as opposed to synchronous generators, induction generators are more favored for renewable energy applications owing to their lower cost and higher reliability. The suitability of using induction generators for power systems application was reported in [2]. In that paper, the authors considered a simple test system consisting of an induction generator interfaced to the utility. Then, the effect of induction machine parameters, modeling requirements and voltage support requirements on the stability of the test system was investigated by numerical simulations. In [3], the impact of wind turbines on steady state security was studied by the methodology of load flow. In that paper, the authors used steady state models (PQ and RX models) to incorporate the induction machine in to the load flow analysis. In [4], the authors developed a linear dynamic model based on [5] for dynamic studies involving wind turbines. It must be remarked that simple steady state models such as the PQ and RX models are used for studying steady state security issues and the dynamic models (as in [2], [4] and [5]) are often used to conduct dynamic studies. For a preliminary analysis of grid connected wind energy conversion systems employing induction generators, one is justified in studying a simple, but representative system as shown in Fig.1 where C represents the capacitor bank at the terminals, x_e represents the transmission line impedance and P_m denotes the mechanical input power extracted from the rotors of the wind turbine in to the induction generator. There are several factors that determine the connection of induction generators to the utility network. In this paper, we focus on three steady state issues that warrant study namely,

A size of the capacitor bank : in traditional schemes employing cage rotor induction machines, the capacitor bank is normally standardized and based on achieving unity power factor operation at nominal power and voltage.

B the maximum electrical power (P_e) that the machine can safely deliver to the grid without endangering system stability.

C grid strength (x_e) : the weakest grid to which the machine can deliver a specified amount of power without endangering system stability, or safe voltage margins.

In determining the issues (B) and (C) listed above, the traditional approach has been the use of numerical techniques along with power system simulation. In this paper, we propose an alternative approach to determine the grid interconnection issues (A, B and C). A third order dynamic model ([2],[5]) is used to represent the induction machine and then, the behavior of the system in steady state is studied simply by analysis of the equilibrium solutions of the resultant nonlinear dynamic system equations. Specifically, the issues A, B and C are addressed as follows.

1) Suppose a certain input mechanical torque to the machine and the external reactance of the transmission line are specified. What is the minimum capacitance of the compensator bank that is required to sustain the resultant equilibrium or steady state ? What is the impact of capacitor size variation on the equilibrium ?
2) Given an *a priori* value of the capacitance of the compensator bank and a certain external reactance, what is the maximum real power that the generator can deliver to the network ?
3) Given an *a priori* value of the capacitance of the compensator bank and a specified input torque to the machine, what is the maximum value of the external reactance (i.e. the weakest network) to which the machine can deliver the corresponding electrical power ?

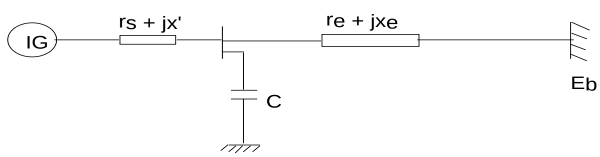

Fig. 1. Schematic system representation

After setting up the dynamic machine equations, we first derive a condition that guarantees the existence of an equilibrium (or steady state) solution to the dynamic system. On further analysis of this condition, we shall derive three auxiliary conditions to study the issues discussed earlier. The rest of this paper is organized as follows. In Sec.2, a dynamic model of the induction generator is used to arrive at the system equations. In Sec.3, the system equations are analyzed to derive a criterion that guarantees the existence of an equilibrium. Further, this criterion is used to arrive at the limiting conditions for each of the issues addressed in the list above. In Sec.4, the effect of variation in capacitance is studied on (i) terminal voltage regulation, (ii) maximum external reactance and (ii) maximum electrical power. Finally, the conclusions are summarized in Sec.5.

II. System Model

We shall represent the induction machine by a third order dynamic model as described in [5]. In this model, we shall consider the transient effects, but neglect the subtransient effects of the rotor. Also, the model assumes a balanced network and neglects the electromagnetic dynamic effects of the stator. Further, we shall reference all quantities to a synchronously rotating frame. Then, the machine can be modeled as an equivalent voltage source E' behind an impedance $r_s + jx'$. The dynamic equations associated with E' are then given by

$$\frac{dE'}{dt} = -js\omega_s E' - \frac{1}{T'_o}(E' - j(x_o - x')I_s) \tag{1}$$

It must be remarked that the model used above is widely used for transient stability studies and analysis of dynamic phenomena involving induction motors and generators (see [4] and [6]). The armature (or stator current) is obtained from

$$U - E' = (r_s + jx')I_s \tag{2}$$

The mechanical equation governing the inertial dynamics of the rotor is given by ([6])

$$\frac{P_m}{1-s} - P_e = -2H\frac{ds}{dt} \tag{3}$$

The circuit equivalent of the system obtained by connecting the induction generator to the infinite bus through an external impedance is shown in Fig.2.

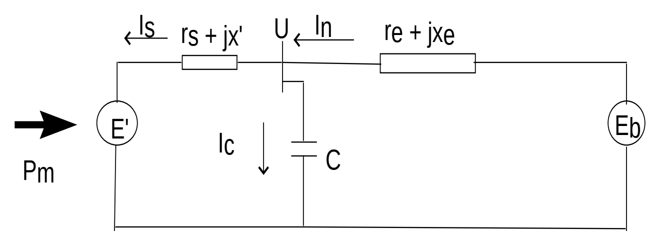

Fig. 2. Equivalent circuit model

Let us denote the following quantities.

- $E_r = Re\{E'\}$, $E_m = Im\{E'\}$ so that $E' = E_r + jE_m$.
- $I_r = Re\{I_s\}$, $I_m = Im\{I_s\}$ so that $I_s = I_r + jI_m$
- $U_r = Re\{U\}, U_m = Im\{U\}$ so that $U = U_r + jU_m$

Neglecting the stator transients, the network equation can be written as

$$\frac{E_b - U}{r_e + jx_e} - UY_c = I_s = \frac{U - E'}{r_s + jx'} \tag{4}$$

From Eq.4, the terminal voltage U can be expressed in terms of E' as follows.

$$\begin{bmatrix} U_r \\ U_m \end{bmatrix} = \frac{1}{D}\begin{bmatrix} a_{11} & a_{12} \\ -a_{12} & a_{11} \end{bmatrix}\begin{bmatrix} b'_1 \\ b'_2 \end{bmatrix} \tag{5}$$

where $D = a_{11}^2 + a_{12}^2$. Substituting Eq.(5) in to Eq.(4), we can solve for the current I_s in terms of E_r, E_m and s. The constants a_{11}, a_{12}, b'_1 and b'_2 are dependent on the network and the machine parameters. The expressions for these constants are presented in Appendix II. The electrical power developed by the machine (P_e) can now be expressed as

$$P_e = -Re\{E' I_s^*\} \tag{6}$$

Upon substituting for I_s and P_e equations (1) and (3) can be expressed as

$$\Sigma: \quad \dot{E}_r = \omega_s s E_m - E_r b_2 + a_3 \tag{7}$$
$$\dot{E_m} = -E_m b_2 - \omega_s s E_r \tag{8}$$
$$\dot{s} = a_4 E_m - \frac{P_m}{2H(1-s)} \tag{9}$$

where a_3, a_4 and b_2 are constants as defined in Appendix B. Equations (7), (8) and (9) describe the dynamical equations of the induction generator-infinite bus system. For brevity, let us denote the system above by Σ. Now, the steady state behavior of the entire system can be simply studied by looking at the equilibrium points of Σ. In the following section, we shall analyze Σ to seek conditions which determine the existence of equilibrium points.

III. When does Σ possess feasible solutions ?

In this section, we shall first seek a criterion that guarantees the existence of an equilibrium point for Σ. The key idea is to obtain a solution for the slip of the machine in terms of the network parameters, the physical machine parameters and the input mechanical power to the machine. Upon setting the derivatives (i.e. the RHS of Eqns.(7), (8) and (9)) to zero, we can explicitly solve for the equilibrium of Σ in terms of the network parameters and the input mechanical power P_m. The solution for the slip s is obtained from a quadratic equation. Then, it is evident that the system Σ possesses an equilibrium if and only if the slip s is real. This requirement and the associated condition is summarized in the following Proposition. Later, we shall use the criterion in this Proposition to obtain the limiting conditions namely, the minimum capacitance, maximum deliverable power and maximum external reactance as explained in section 1.

Proposition 1 Σ possesses a feasible equilibrium if and only if

$$\Delta = \alpha_2^2 - 4\alpha_1\alpha_3 \geq 0 \tag{10}$$

where,
$\alpha_1 = a_2 b_2 P_m \omega_s^2 - w_s E_b a_3 b_2$
$\alpha_2 = \omega_s E_b a_3 b_2$ and $\alpha_3 = a_2 b_2 P_m b_2^2$.

Proof: Upon setting the derivatives of equations (7), (8) and (9) to zero, we obtain a solution for the slip s in the form of a quadratic which can be expressed as

$$s^2\alpha_1 + s\alpha_2 + \alpha_3 = 0 \tag{11}$$

It is clear the the system can have a feasible solution if the slip is real and vice-versa. The claim in the proposition merely asserts the non-negativity of the discriminant of the quadratic in Eq.(11) so that the system Σ possesses real solutions. ■

Note: The parameters a_2, b_2 and a_3 and hence, α_1, α_2 and α_3 are functions of the network parameters x_e, C and the input mechanical power P_m. This in turn means that the equilibrium and consequently, its existence is a function of the triple (C, x_e, P_m). If any two quantities in the triple are held fixed, the equilibrium moves as we vary the third. In the analysis that follows, we shall exploit this dependence to arrive at suitable limiting conditions for the variation of each of the quantities in the triple.

A. Minimum Capacitance

In this section, we shall compute the minimum value of the capacitance at the terminals of the machine, by using the criterion in Proposition 1. Specifically, we seek to answer the following question "Suppose that the mechanical input power to the machine P_m and the external reactance of the system x_e are specified. What is the minimum value of C that ensures that an equilibrium exists for Σ ?" The answer follows readily by using the criterion (Eqn.(10)) in Proposition 1. We shall state this result as described in the following proposition.

Proposition 2 : Given the input power $P_m > \kappa$ and $x_e > 0$, an equilibrium to Σ exists if

$$C \geq C_{min} = \frac{x_e + x' - \beta_{min}}{x_e x' \omega_s} \tag{12}$$

where β_{min} is the least positive real root of the polynomial

$$x^4\beta_1 + x^3\beta_2 + x^2\beta_3 + x\beta_4 + \beta_5 = 0 \tag{13}$$

and $\kappa = \frac{E_b^2 x_o^2}{\omega_s x_a x_e^2}$.

Proof: First, we shall express the discriminant of Eqn.(11) in terms of a_2. After substituting for a_2, a_3 and b_2 from Appendix B in to equation (11), the condition $\Delta > 0$ is tantamount to requiring that

$$f_1(a_2) = a_2^4\beta_1 + a_2^3\beta_2 + a_2^2\beta_3 + a_1\beta_4 + \beta_5 < 0 \tag{14}$$

where,

$$\beta_1 = 4P_m^2 w_s^2 x_o^2$$
$$\beta_2 = -8P_m^2 w_s^2 x_o x_a x_3$$
$$\beta_3 = 4P_m\omega_s x_a(P_m\omega_s x_a x_e^2 - \alpha_o E_b^2 x_o^2)$$
$$\beta_4 = 8P_m\omega_s x_e x_o x_a^2 \alpha_o E_b^2$$
$$\beta_5 = -4P_m\omega_s x_a^3 x_e^2 E_b^2 - {x'}^2 E_b^4 x_a^2$$

Observe that $\beta_1 > 0, \beta_2 < 0, \beta_4 > 0$and $\beta_5 < 0$. The sign of β_3 depends on the value of P_m. Let $\kappa = \frac{E_b^2 x_o^2}{\omega_s x_a x_e^2}$. When the machine parameters are fixed, the value of κ depends only on the external reactance x_e. In the range of x_e that we consider, which is $0.2 < x_e < 0.8$ and for the machine parameters we consider (see Appendix I), the corresponding range of κ is $0.01 < \kappa < 0.16$. Then it is reasonable to consider $P_m > \kappa$ in the normal operating range of the generator. Now, consider the roots of the polynomial $f_1(a_2) = 0$. Given the signs of the coefficients $\beta_1, \ldots \beta_5$, a simple application of the Routh-Hurwitz criterion indicates that for the polynomial $f_1(a_2) = 0$, there are three roots whose real parts are positive. Let us denote these roots as $\lambda_1, \lambda_2, \lambda_3$. Accordingly, we have two cases.

case a: all three roots are entirely real.
case b: one root (say λ_1) is real and the other two are complex conjugate (i.e. $\lambda_2 = {\lambda_3}^*$).

In the first case (a), let $\beta_{min} = \min\{\lambda_1, \lambda_2, \lambda_3\}$. For the second case (b), let $\beta_{min} = \lambda_1$. Clearly, if $a_2 < \beta_{min}$, then,

$f_1(a_2) < 0$ which implies $\Delta > 0$ which in turn, assures the existence of an equilibrium. So we obtain,

$$a_2 = x_e + x' - Y_C x_e x' < \beta_{min} \tag{15}$$

Then, the claim in the proposition readily follows from Eqn.(15). ■

B. Maximum deliverable power

In this section, we shall compute the theoretical maximum power that the induction generator can deliver to a given external reactance. Specifically, suppose we are given the capacitance of the compensator bank and the reactance of the transmission network, then what is the maximum value of P_m beyond which an equilibrium to Σ does not exist. Again, this can be readily answered by the criterion in Proposition 1. In this analysis, note that we hold C and x_e fixed. The result is summarized in the following proposition.

Proposition 3 : Given the input power P_m and $x_e > 0$, the maximum real power that the machine can deliver to the network is given by

$$P_{max} = \frac{E_b a_3}{4\omega_s a_2}[2 + \sqrt{1 + \frac{4\omega_s^2}{b_2^2}}] \tag{16}$$

Proof: For an equilibrium to exist, we need (from Proposition 1, Eqn.(10)) $\Delta > 0$. Substituting for all the terms in Δ, we get

$$\Delta = b_2^2\{\omega_s^2 E_b^2 a_3^2 - 4a_2^2 b_2^2 P_m^2 \omega_s^2 + 4a_2 b_2^2 E_b a_3 P_m\} \tag{17}$$

Clearly, $\Delta > 0$ implies that,

$$\begin{aligned} f_2(P_m) &= P_m^2(4a_2^2 b_2^2 \omega_s^2) - P_m(4a_2 b_2^2 \omega_s E_b a_3) - \omega_s E_b^2 a_3^2 \\ &= (P_m - \gamma_1)(P_m - \gamma_2) < 0 \end{aligned} \tag{18}$$

Note that we have labelled the two roots of the equation $f_2(P_m) = 0$ as γ_1 and γ_2 where $\gamma_1 > 0$ and $\gamma_2 < 0$. Then, it is clear that the maximum value of P_m is decided by γ_1. Thus we have,

$$P_{max} = \gamma_1 = \frac{E_b a_3}{4\omega_s a_2}[2 + \sqrt{1 + \frac{4\omega_s^2}{b_2^2}}] \tag{19}$$

■

C. Maximum external reactance

In this section, we shall assume that we are given the mechanical input power (P_m) and the value of the capacitance (C) at the machine terminals. Then, the question we wish to answer is, "what is the maximum value of the external reactance (which we shall denote as x_e^{max}) to which the machine can deliver the specified power ?". This can be readily answered by rewriting the term Δ in Proposition 1 as a function of x_e. After a few algebraic manipulations, Δ can be arranged as a polynomial (6'th order) in x_e. Then the criterion for the maximum value of x_e is obtained from the following proposition.

Proposition 4 : Given the input power P_m and C, the maximum external reactance to which the machine can deliver power is given by

$$x_e^{max} = \eta \tag{20}$$

where η is the largest positive real root of the polynomial

$$f_3(x_e) = x_e^6\eta_1 + x_e^5\eta_2 + x_e^4\eta_3 + x_e^3\eta_4 + x_e^2\eta_5 + x_e\eta_6 + \eta_7 = 0 \tag{21}$$

The terms $\eta_i, i = 1, 2 \ldots 7$ are as described in the appendix.

Proof: The condition $\Delta > 0$ from Proposition 1 is equivalent to Eqn.(17) in proposition 3. Expressing the terms a_2, b_2 and a_3 in terms of x_e yields

$$\Delta = -b_2^4\{x_e^6\eta_1 + x_e^5\eta_2 + x_e^4\eta_3 + x_e^3\eta_4 + x_e^2\eta_5 + x_e\eta_6 + \eta_7\} \tag{22}$$

Note that if $f_3(x_e) < 0$, then $\Delta > 0$. The condition $f_3(x_e) < 0$ can be clearly met if $x_e < \eta$ which is the claim in this proposition. ■

Note : In this case (as opposed to propositions 1 and 2), the algebraic form of the coefficients $\eta_1, \ldots \eta_7$ are complicated which makes it hard to analyze the signs of these coefficients and deduce the structure of the roots of the polynomial $f_3(x_e) = 0$.

Remarks :

1) In proposition 2, we assume P_m and x_e as specified (i.e. fixed) and calculate the minimum capacitance C_{min} as a function of P_m and x_e.
2) In proposition 3, we assume that C and x_e as specified and calculate the maximum power that P_m^{max} that the machine can deliver to the network.
3) In proposition 4, we assume that P_m and C as specified and calculate the maximum external reactance x_e to which the machine can deliver the specified power.
4) Note that all the three propositions directly follow from Proposition 1. The three propositions explicitly characterize the functional dependence of C, P_m and x_e for existence of equilibrium points when any two of them are held fixed. In the next section, we study the effect of capacitance on the terminal voltage regulation. And finally, we numerically study how the minimum required capacitance, maximum deliverable power and maximum reactance vary as a function of the their respective parameters.

IV. Voltage regulation and its consequences

In the analysis carried out so far, we have not investigated the bearing of the various parameters namely C, x_e and P_m on the terminal voltage (U) of the machine. In this section, we shall study the effect of the capacitance (C) on the terminal voltage U for various values of the external reactance and input power P_m. Not surprisingly, the dependence of the terminal voltage on the capacitance is strong. Next, we impose a reasonable operational constraint that $0.95 \leq U \leq 1.05$ which allows a $\pm 5\%$ variation of the steady state terminal voltage around the nominal value of

1.0 p.u. This constraint then effectively yields the allowable or permissible range for the variation in the capacitance. Having obtained the effective range for the capacitance, we can obtain the corresponding ranges for the maximum transferable power P_{max} and x_e^{max}, the maximum external reactance to which the machine can deliver power. This is done as explained below.
After solving for the equilibrium points as described in the appendix, we can write

$$U_r = \frac{E_r x_e + E_b x'}{a_2}\ , U_m = \frac{E_m x_e}{a_2} \tag{23}$$

$$|U| = \sqrt{U_r^2 + U_m^2} \tag{24}$$

The variation in terminal voltage U with capacitance is plotted for various values of the external reactance. Figure 3 shows a plot of the terminal voltage when P_m is held constant at 0.9 p.u for $x_e = 0.2, 0.3, 0.4$ and 0.5. Similarly, Fig. 4 shows the terminal voltage variation when $P_m = 1.0$. In both the plots, we observe that the terminal voltage rises steeply with capacitance and more so, at higher values of the external impedance x_e. Now, imposing the constraint that $0.95 < |U| < 1.05$ naturally imposes a suitable constraint (in other words, restricts the range) on the capacitance C. For a given value of P_m and x_e, let us define C_{low} as the capacitance required to maintain the terminal voltage magnitude U at 0.95 p.u and C_{hi} as the corresponding value to maintain the terminal voltage at 1.05 p.u. When $P_m = 0.9$and 1.0, the capacitive limits C_{low}, C_{hi} (expressed in 10^{-3} p.u) for various values of the line impedance are summarized in Table.I. As one can note from Table.I, the effective range of capacitance (=$C_{hi} - C_{low}$) shrinks quickly with increasing values of the external reactance. When $x_e = 0.2$, we get a capacitive range of 0.5$\times 10^{-3}$ p.u which reduces to 0.05$\times 10^{-3}$ p.u when $x_e = 0.5$.

TABLE I
SUMMARY OF CAPACITIVE RANGE FOR VOLTAGE REGULATION

line reactance	$P_m = 0.9$		$P_m = 1.0$	
	C_{low}	C_{hi}	C_{low}	C_{hi}
$x_e = 0.2$	1.5	2	1.8	2.2
$x_e = 0.3$	1.7	2	2.0	2.2
$x_e = 0.4$	1.9	2.1	2.2	2.3
$x_e = 0.5$	2.1	2.15	2.3	2.4

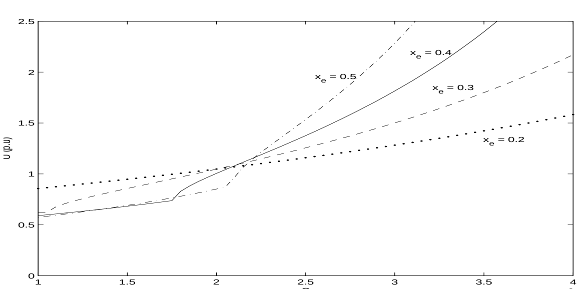

Fig. 3. Terminal voltage variation when $P_m = 0.9$

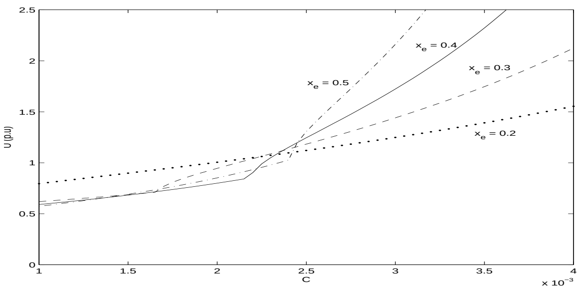

Fig. 4. Terminal voltage variation when $P_m = 1.0$

A. Effect of capacitance on P^{max} and x_e^{max}

In this section, we shall graphically study the effect of capacitance on the maximum deliverable power P^{max} and the maximum external reactance x_e^{max}. This is done mainly because it is hard to gauge this dependence directly from Eqns.(16) and (20). In sections III-B and III-C, we derived analytical expressions for P^{max} (see Eq. 16) and x_e^{max} (see Eqn. 20). Figures 5 and 6 show the variation in P^{max} and x_e^{max} with the capacitance C. From Fig.5, we note that when $x_e = 0.2$, P^{max} stays consistently above 1.0 p.u and is about 1.7 p.u when the capacitance is 4.5×10^{-3}. Also notice that the capacitive requirements to maintain a real power output of 1.0 p.u go up with increasing values of the line reactance. Fig.6 shows how the maximum external reactance to which the machine can deliver a specified real power, varies with the capacitance. From Fig.6, we note that when $P_m = 0.8$, x_e^{max} rises rather sharply to a maximum of about 0.7 p.u when the capacitance C is nearly 4×10^{-3} p.u. When $P_m = 0.9$ and 1.0 p.u, for nearly the same value of capacitance, x_e^{max} is about 0.62 and 0.57 respectively.

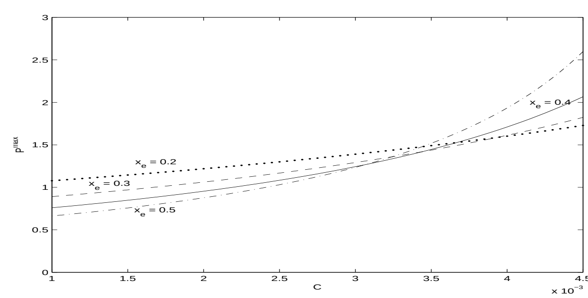

Fig. 5. Effect of C on P^{max}

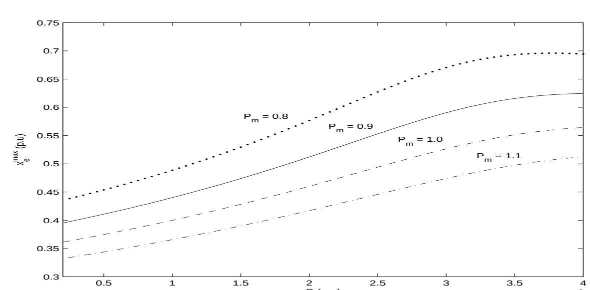

Fig. 6. Effect of C on x_e^{max}

B. Effect of line reactance on C_{min}

In Fig.7, the minimum capacitance (C_{min}) is plotted as a function of the line reactance x_e for different values of the input power P_m. Note that C_{min} is computed from Eq.(12). In the plot, C_{min} is set to zero whenever Eq.(12) yields a negative value of C_{min}. From Fig.7, we see that at lower values of the line impedance x_e, capacitive compensation is not required for an equilibrium to exist. As the line reactance is increased for a fixed mechanical power input, capacitive compensation is required beyond a critical value of the line reactance. From the plot, it is clear that the critical line reactance reduces as the input power P_m is increased.

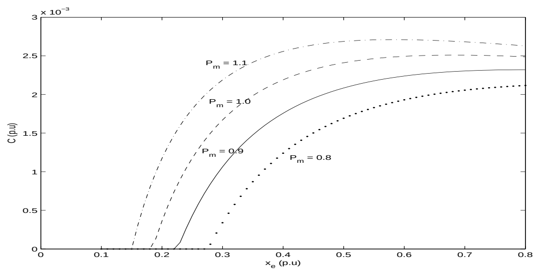

Fig. 7. Variation of C_{min}

C. Discussion

1) In Fig.3, we observe that for the different values of x_e ranging from 0.2 to 0.5 p.u, the curves roughly seem to intersect at a point when the terminal voltage is slightly higher than 1.0 p.u. From Table.I and Fig.3, this point is approximately seen to be 2.1 $\times 10^{-3}$ p.u. A similar feature is noticed in Fig.4, when $P_m = 1.0$. In this case, from Fig.4 and Table.I, the value of capacitance is seen to be approximately 2.2 $\times 10^{-3}$ p.u. These observations indicate that the task of steady state terminal voltage regulation with changing external line reactance can be achieved with minimum capacitive switching effort if the capacitance of the compensator is set close to this value.
2) In Fig.5, we notice that sustaining a power output bigger than 1.0 p.u from the machine at higher values of the line reactance requires prohibitively high values of capacitive compensation. This is because we get prohibitively high values for the terminal voltage at these high values of capacitance.
3) In Fig.7, we notice that beyond a critical line reactance, the minimum capacitive requirements go up steeply at first, with increasing values of the line reactance. Observe that the curves saturate when the line reactance is about 0.6 p.u. For an increase in line reactance beyond 0.6 p.u, the corresponding growth in C_{min} diminishes rapidly.

V. CONCLUSIONS

A system comprising of a wind energy converter (WEC) connected to the utility through a transmission line is studied. An induction generator employing a capacitive compensator bank is used to model the WEC. A third order (nonlinear) dynamic model is used to represent the induction generator. The resultant system equations are analyzed to derive a criterion which ensures the existence of an equilibrium or steady state solution to the system. This criterion is further analyzed to compute (i) the minimum value of the capacitance of the compensator bank to deliver a specified value of real power to the given network, (ii) the maximum real power deliverable by the machine to the network and (iii) the maximum external reactance to which the machine can deliver power. This approach is seen to provide useful analytical insight in to the operation of an induction generator connected to an infinite bus. A numerical study of the effect of capacitance on terminal voltage regulation indicates that terminal voltage regulation has the greatest limiting influence on the power transfer and the weakest transmission line that can convey the power to the infinite bus. Thus, the analysis presented could be a useful tool for preliminary planning studies involving wind energy converters.

APPENDIX I
INDUCTION GENERATOR DATA

Base voltage = 660 V, Base power = 350 kVA, $x_r = 0.0639$p.u, $x_s = 0.1878$ p.u, $r_r = 0.00612$ p.u $r_s = 0.00571$p.u, $x_m = 2.78$ p.u, $H = 3.025s$, $\omega_s = 120\pi$ rad/s, $E_b = 1.0$p.u
The machine constants T_o', x' and x_o are defined as follows:
$T_o' = \frac{x_r + x_m}{\omega_s r_r}, x_o = x_r + x_m, x' = x_s + \frac{x_r x_m}{x_r + x_m}$

APPENDIX II
COEFFICIENTS AND PARAMETERS

In Eqn.(5), the parameters are described as follows.

$$a_{11} = r_e + r_s - x' Y_c r_e - Y_c r_s x_e$$
$$a_{12} = x_e + x' + Y_c r_s r_e - Y_c x' x_e$$
$$b_1' = E_r r_e - E_m x_e + E_b r_s$$
$$b_2' = E_m r_e + E_r x_e + E_b x'$$

In Eqns.(7), (8) and (9), the parameters are described as follows.

$$a_2 = x_e + x' - Y_c x_e x', \;\; b_1 = \frac{a_2 - x_e}{a_2 x'}, \;\; x_a = x_o - x'$$
$$b_2 = \alpha_o (1 + x_a b_1) \;, \alpha_o = \frac{1}{T_o'}, \;\; a_3 = \frac{\alpha_o x_a E_b}{a_2}, \;\; a_4 = \frac{E_b}{2Ha_2}$$

In proposition 3, the coefficients $\eta_1, \ldots \eta_7$ are defined as follows.

$$\eta_1 = K_1 c^4 b^2, \;\; \eta_2 = -2abc^4 K_1 + 4c^2 x' b^2 K_1$$
$$\eta_3 = K_1 a^2 c^4 + 4{x'}^2 b^2 K_1 + 2c^2 {x'}^2 b^2 K_1 - 8abc^2 x' K_1 - K_2 b^2 c^2$$
$$\eta_4 = 4c^2 a^2 x' K_1 + 4{x'}^3 b^2 K_1 - 4abc^2 {x'}^2 K_1 - 8{x'}^2 ab + 2abc^2 K_2$$
$$\eta_5 = 4{x'}^2 a^2 K_1 + 2c^2 a^2 {x'}^2 K_1 - 8ab{x'}^3 K_1 + {x'}^4 b^2 K_1 - K_2 a^2 c^2 - b^2 {x'}^2 K_2 + 4abcx' K_2$$
$$\eta_6 = -2ab{x'}^4 K_1 - 2cx' a^2 K_2 + 2ab{x'}^2 K_2$$
$$\eta_7 = K_1 {x'}^4 a^2 - K_2 {x'}^2 a^2$$

where,

$$a = \alpha_o(1 + x_a), \;\; b = \alpha_o Y_c x_e, \;\; c = 1 - Y_c x', \;\; K_1 = 4P_m^2\omega_s^2$$
$$K_2 = 4P_m\omega_s\alpha_o x_a E_b^2$$

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