STEADY STATE ANALYSIS OF INDUCTION GENERATOR INFINITE BUS SYSTEMS

Rajesh G. Kavasseri
Department of Electrical and Computer Engineering
North Dakota State University, Fargo, ND 58105 - 5285, USA.
(email: rajesh.kavasseri@ndsu.nodak.edu)

Abstract : This paper conducts a fundamental analysis on the induction generator infinite bus system which is a useful representation for a wind energy converter interfaced to a utility through a transmission line. A third order model is used to represent the induction generator and the resultant nonlinear system equations are analysed to derive a condition that guarantees the existence of equilibrium points (or steady state solutions) to the dynamic system. This condition is used to derive three other auxiliary conditions which compute (i) the minimum value of capacitance, (ii) the maximum deliverable power and (iii) the maximum external reactance that can be connected to the machine. The conditions listed above are further studied numerically on three different sets of induction generator data. The impact of capacitive requirements on power factor and terminal voltage is examined. It is shown that terminal voltage regulation has a strong restrictive influence on the maximum transferable power. The analysis and studies presented could be a useful tool for preliminary planning studies involving wind energy converters.

Key Words : induction generators, wind energy conversion

Nomenclature

- r_s : armature (stator) resistance
- x_1 : armature leakage reactance
- x_m : magnetizing reactance
- x_2 : rotor leakage reactance
- r_r : rotor resistance
- ω_s : angular speed of the synchronously rotating frame
- s : slip of the machine
- E' : equivalent voltage source of the machine

- I_s : armature current of the machine
- U : terminal voltage of the machine
- T_o' : rotor open circuit time constant
- x' : transient reactance
- P_e : electrical power output of the machine
- P_m : input mechanical power to the machine
- H : inertia constant
- I_c : current through the compensator bank
- E_b : magnitude of the infinite bus voltage
- r_e : resistance of the transmission line
- x_e : reactance of the transmission line
- Y_c: admittance of the capacitor bank

1 Introduction

Wind energy conversion has emerged as a viable alternative to meet the increased demand for energy resources in recent years. There are different configurations for wind turbine units which include synchronous, asynchronous generators, pitch regulated, or stall regulated systems. Broadly speaking, wind energy conversion systems can be classified in to variable speed and fixed speed devices. Variable speed systems employ an electronic interface for grid connection. Fixed speed systems commonly use squirrel cage induction generators which are directly connected to the grid. While the former is better suited for optimal energy extraction attractive option, the latter is simpler and cheaper and therefore, a considerable number of such systems are currently in operation. For instance, most of the Spanish wind energy is supplied by fixed speed turbines rated between 500 and 1000 kW clustered in wind farms ranging from 30 to 250 MW, [1]. Therefore, the analysis of fixed speed systems can be important, especially when several of these are installed and continue to remain in operation. In fixed speed systems, induction generators are particularly favoured owing to their lower cost and higher reliability. The suitability of using induction generators for power systems application was reported in [2]. In that paper, the authors considered a simple test system consisting of an induction generator interfaced to the utility. The effect of induction machine parameters, modelling requirements and voltage support requirements on the stability of the test system was investigated by numerical simulations. In [3], the impact of wind turbines on steady state security was studied by the methodology of load flow using the PQ and RX models which are approximations of the steady state induction machine model. In this paper, a third order model is used for the induction machine and the resultant system equations are directly analysed to study the steady state operation of the system.

We consider a system which consists of a wind generation facility (employing squirrel cage induction generators) interfaced to the utility, or grid, through a transmission line as shown in Fig.1. A large wind farm can consist of hundreds of wind turbines connected by a series of feeders. If one is interested in studying the performance of the wind farm in itself, it will be required to represent each generator individually, [5]. However, for system impact studies where it is required to study the interaction of a whole wind farm with the rest of the system, a detailed model of individual generators would result in very cumbersome calculations, [5]. In such cases, it would be advantageous to represent the wind farm by an equivalent model. In this paper, we consider fixed speed systems where the speed deviations between generators in a wind farm are minor and therefore, the electrical equivalent of the wind farm can be approximated by one equivalent induction generator, [5]. A study in [5] also shows that the aggregated wind farm model with just one induction generator can reproduce the combined effect of power fluctuations (due to non-identical wind speeds) at the individual turbines locations with good accuracy. As the present paper is aimed solely at analysing the steady state electrical performance of the grid- interfaced wind farm system, it is reasonable to use the representation shown in Fig.1 where C represents the capacitor bank at the terminals for reactive power compensation, $z_e = r_e + jx_e$ represents the transmission line impedance and E_b is the voltage of the infinite bus which represents the grid. This representation facilitates both analysis/simulation studies with fixed speed wind farms and examples can be found in [6] and [7]. While there are several factors that determine the connection of wind farms to the grid, we focus on three steady state issues that warrant study namely,

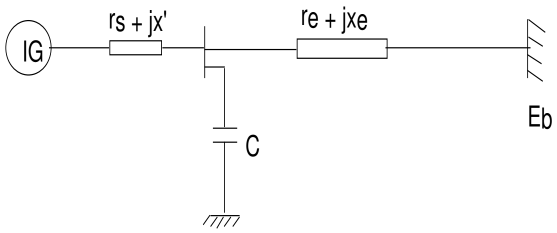

Figure 1: Schematic system representation

A size of the capacitor bank : in traditional schemes employing cage rotor induction machines, the capacitor bank is normally standardized and based on achieving unity power factor operation at nominal power and voltage.

B the maximum electrical power (P_e) that the machine can safely deliver to the grid without endangering system stability.

C grid strength (x_e) : the weakest grid to which the machine can deliver a specified amount of power without endangering system stability, or safe voltage margins.

In determining the issues (B) and (C) listed above, the traditional approach has been the use of numerical techniques along with power system simulation. In this paper, we propose an alternative approach to determine the grid interconnection issues listed above. A third order dynamic model ([2],[8]) is used to represent the induction machine and the behaviour of the system in steady state is studied simply by analysis of the equilibrium solutions of the resultant nonlinear dynamic system equations. Specifically, the issues A, B and C are addressed as follows.

1. Suppose a certain input mechanical torque to the machine and the external reactance of the transmission line are specified. What is the minimum capacitance of the compensator bank that is required to sustain the resultant equilibrium or steady state ? What is the impact of capacitor size variation on the equilibrium ?

2. Given an *a priori* value of the capacitance of the compensator bank and a certain external reactance, what is the maximum real power that the generator can deliver to the network ?

3. Given an *a priori* value of the capacitance of the compensator bank and a specified input torque to the machine, what is the maximum value of the external reactance (i.e. the weakest network) to which the machine can deliver the corresponding electrical power ? This is particularly important for grid integration studies involving wind energy systems with weak interconnection ties.

A preliminary study of these issues was presented in [9], where the analysis was done on data pertaining to a small induction generator. In this paper, the study is extended to analyse three different sets of induction generator data. In addition, the influence of these issues on the power factor of the machine is studied in greater detail. The rest of this paper is organized as follows. In Sec.2, a dynamic model of the induction generator is used to arrive at the system equations. In Sec.3, the system equations are analysed to derive a criterion that guarantees the existence of an equilibrium. Further, this criterion is used to arrive at the limiting conditions for each of the issues addressed in the list above. In Sec.4, the effect of variation in capacitance is studied on (i) terminal voltage regulation, (ii) power factor, (iii) maximum external reactance and (iv) maximum electrical power. The conclusions are summarised in Sec.6.

2 System Model

We shall represent the induction machine by a third order dynamic model as described in [8]. In this model, we shall consider the transient effects, but neglect the subtransient effects of the rotor. Also, the model assumes a balanced network and neglects the electromagnetic dynamic effects of the stator. Further, we shall reference all quantities to a synchronously rotating frame. Then, the machine can be modeled as an equivalent voltage source E' behind an impedance $r_s + jx'$. The dynamic equations associated with E' are then given by, [4], [10]

$$\frac{dE'}{dt} = -js\omega_s E' - \frac{1}{T'_o}(E' - j(x_o - x')I_s) \tag{1}$$

The armature (or stator current) is obtained from

$$E' - U = (r_s + jx')I_s \tag{2}$$

The mechanical equation governing the inertial dynamics of the rotor is given by ([10])

$$\frac{P_m}{1-s} - P_e = -2H\frac{ds}{dt} \tag{3}$$

The circuit equivalent of the system obtained by connecting the induction generator to the infinite bus through an external impedance is shown in Fig.2.

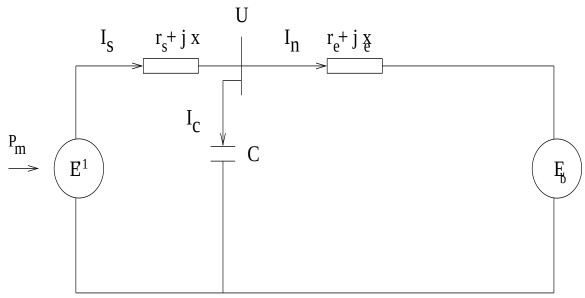

Figure 2: Equivalent circuit model

Let us denote the following quantities.
$E' = E_r + jE_m$, $I_s = I_r + jI_m$ and $U = U_r + jU_m$

Neglecting the stator transients, the network equation can be written as

$$I_s = \frac{E' - U}{r_s + jx'} = \frac{U - E_b}{r_e + jx_e} + UY_c \tag{4}$$

From Eq.(4), the terminal voltage U can be expressed in terms of E' as follows.

$$\begin{bmatrix} U_r \\ U_m \end{bmatrix} = \frac{1}{D}\begin{bmatrix} a_{11} & a_{12} \\ -a_{12} & a_{11} \end{bmatrix}\begin{bmatrix} b_1' \\ b_2' \end{bmatrix} \tag{5}$$

where $D = a_{11}^2 + a_{12}^2$. Substituting Eq.(5) in to Eq.(4), we can solve for the current I_s in terms of E_r, E_m and s. The constants a_{11}, a_{12}, b_1' and b_2' are dependent on the network and the machine parameters. The expressions for these constants are presented in Appendix B. The electrical power developed by the machine (P_e) can now be expressed as

$$P_e = Re\{E'I_s^*\} \tag{6}$$

Upon substituting for I_s and P_e equations (1) and (3) can be expressed as

$$\Sigma: \quad \dot{E_r} = \omega_s s E_m - E_r b_2 + a_3 \tag{7}$$

$$\dot{E_m} = -E_m b_2 - \omega_s s E_r \tag{8}$$

$$\dot{s} = a_4 E_m - \frac{P_m}{2H(1-s)} \tag{9}$$

where a_3, a_4 and b_2 are constants as defined in Appendix B. Equations (7), (8) and (9) describe the dynamical equations of the induction generator-infinite bus system. For brevity, let us denote the system above by Σ. Now, the steady state behaviour of the entire system can be simply studied by looking at the equilibrium points of Σ. In the following section, we shall seek conditions to determine the existence of equilibrium points for Σ.

3 When does Σ possess feasible solutions ?

In this section, we shall first seek a criterion that guarantees the existence of an equilibrium point for Σ. The key idea is to obtain a solution for the slip of the machine in terms of the network parameters, the physical machine parameters and the input mechanical power to the machine. Upon setting the derivatives (i.e. the RHS of Eqns.(7), (8) and (9)) to zero, we can explicitly solve for the equilibrium of Σ in terms of the network parameters and the input mechanical power P_m. The solution for the slip s is obtained from a quadratic equation. Then, it is evident that the system Σ possesses an equilibrium if and only if the slip s is real. This requirement and the associated condition is summarised in the following Proposition. Later, we shall use the criterion in this Proposition to obtain the limiting conditions namely, the minimum capacitance, maximum deliverable power and maximum external reactance as explained in section 1.

Proposition 1 Σ possesses a feasible equilibrium if and only if

$$\Delta = \alpha_2^2 - 4\alpha_1\alpha_3 \geq 0 \tag{10}$$

where,
$\alpha_1 = a_2 b_2 P_m \omega_s^2 - w_s E_b a_3 b_2$
$\alpha_2 = \omega_s E_b a_3 b_2$ and $\alpha_3 = a_2 b_2 P_m b_2^2$.

Proof : Upon setting the derivatives of equations (7), (8) and (9) to zero, we obtain a solution for the slip s in the form of a quadratic which can be expressed as

$$s^2\alpha_1 + s\alpha_2 + \alpha_3 = 0 \tag{11}$$

It is clear the the system can have a feasible solution if the slip is real and vice-versa. The claim in the proposition merely asserts the non-negativity of the discriminant of the quadratic in Eq.(11) so that the system Σ possesses real solutions.

Note: The parameters a_2, b_2 and a_3 and hence, α_1, α_2 and α_3 are functions of the network parameters x_e, C and the input mechanical power P_m. This in turn means that the equilibrium solution is a function of the triple (C, x_e, P_m). If any two quantities in the triple are held fixed, the equilibrium moves as we vary the third. In the analysis that follows, we shall exploit this dependence to arrive at suitable limiting conditions for the variation of each of the quantities in the triple.

3.1 Minimum Capacitance

In this section, we shall compute the minimum value of the capacitance at the terminals of the machine, by using the criterion in Proposition 1. Specifically, we seek to answer the following question "Suppose that the mechanical input power to the machine P_m and the external reactance of the system x_e are specified. What is the minimum value of C that ensures that an equilibrium exists for Σ ?" The answer follows readily by using the criterion (Eqn.(10)) in Proposition 1. We shall state this result as described in the following proposition.

Proposition 2 : Given the input power $P_m > \kappa$ and $x_e > 0$, an equilibrium to Σ exists if

$$C \geq C_{min} = \frac{x_e + x' - \beta_{min}}{x_e x' \omega_s} \tag{12}$$

where β_{min} is the least positive real root of the polynomial

$$x^4\beta_1 + x^3\beta_2 + x^2\beta_3 + x\beta_4 + \beta_5 = 0 \tag{13}$$

with

$$\begin{aligned} \beta_1 &= 4P_m^2 w_s^2 x_o^2 \\ \beta_2 &= -8P_m^2 w_s^2 x_o x_a x_3 \\ \beta_3 &= 4P_m \omega_s x_a (P_m \omega_s x_a x_e^2 - \alpha_o E_b^2 x_o^2) \\ \beta_4 &= 8P_m \omega_s x_e x_o x_a^2 \alpha_o E_b^2 \\ \beta_5 &= -4P_m \omega_s x_a^3 x_e^2 E_b^2 - x'^2 E_b^4 x_a^2 \end{aligned}$$

and $\kappa = \frac{E_b^2 x_o^2}{\omega_s x_a x_e^2}$.

Proof : First, we shall express the discriminant of Eqn.(11) in terms of a_2. After substituting for a_2, a_3 and b_2 from Appendix B in to equation (11), the condition $\Delta > 0$ is tantamount to requiring that

$$f_1(a_2) = a_2^4\beta_1 + a_2^3\beta_2 + a_2^2\beta_3 + a_1\beta_4 + \beta_5 < 0 \tag{14}$$

Observe that $\beta_1 > 0, \beta_2 < 0, \beta_4 > 0$ and $\beta_5 < 0$. The sign of β_3 depends on the value of P_m. Let $\kappa = \frac{E_b^2 x_o^2}{\omega_s x_a x_e^2}$. When the machine parameters are fixed, the value of κ depends only on the external reactance x_e. In the range of x_e that we consider, which is $0.2 < x_e < 0.8$ and for the machine parameters we consider (see Appendix A), the corresponding range of κ is $0.01 < \kappa < 0.16$. Then it is reasonable to consider $P_m > \kappa$ in the normal operating range of the generator. Now, consider the roots of the polynomial $f_1(a_2) = 0$. Given the signs of the coefficients $\beta_1, \ldots \beta_5$, a simple application of the Routh-Hurwitz criterion indicates that for the polynomial $f_1(a_2) = 0$, there are three roots whose real parts are positive. Let us denote these roots as $\lambda_1, \lambda_2, \lambda_3$. Accordingly, we have two cases.

- **case a:** all three roots are entirely real.
- **case b:** one root (say λ_1) is real and the other two are complex conjugate (i.e. $\lambda_2 = \lambda_3{}^*$).

In the first case (a), let $\beta_{min} = \min\{\lambda_1, \lambda_2, \lambda_3\}$. For the second case (b), let $\beta_{min} = \lambda_1$. Clearly, if $a_2 < \beta_{min}$, then, $f_1(a_2) < 0$ which implies $\Delta > 0$ which in turn, assures the existence of an equilibrium. So we obtain,

$$a_2 = x_e + x' - Y_C x_e x' < \beta_{min} \tag{15}$$

Then, the claim in the proposition readily follows from Eqn.(15).

3.2 Maximum deliverable power

In this section, we shall compute the theoretical maximum power that the induction generator can deliver to a given external reactance. Specifically, suppose we are given the capacitance of the compensator bank and the reactance of the transmission network, then what is the maximum value of P_m beyond which an equilibrium to Σ does not exist. Again, this can be readily answered by the criterion in Proposition 1. In this analysis, note that we hold C and x_e fixed. The result is summarised in the following proposition.

Proposition 3 : Given the input power P_m and $x_e > 0$, the maximum real power that the machine can deliver to the network is given by

$$P_{max} = \frac{E_b a_3}{4\omega_s a_2}[2 + \sqrt{1 + \frac{4\omega_s^2}{b_2^2}}] \tag{16}$$

Proof : For an equilibrium to exist, we need (from Proposition 1, Eqn.(10)) $\Delta > 0$. Substituting for all the terms in Δ, we get

$$\Delta = b_2^2\{\omega_s^2 E_b^2 a_3^2 - 4a_2^2 b_2^2 P_m^2 \omega_s^2 + 4a_2 b_2^2 E_b a_3 P_m\} \tag{17}$$

Clearly, $\Delta > 0$ implies that,

$$\begin{aligned} f_2(P_m) &= P_m^2(4a_2^2 b_2^2 \omega_s^2) - P_m(4a_2 b_2^2 \omega_s E_b a_3) - \omega_s E_b^2 a_3^2 \\ &= (P_m - \gamma_1)(P_m - \gamma_2) < 0 \end{aligned} \tag{18}$$

Note that we have labelled the two roots of the equation $f_2(P_m) = 0$ as γ_1 and γ_2 where $\gamma_1 > 0$ and $\gamma_2 < 0$. Then, it is clear that the maximum value of P_m is decided by γ_1. Thus we have,

$$P_{max} = \gamma_1 = \frac{E_b a_3}{4\omega_s a_2}[2 + \sqrt{1 + \frac{4\omega_s^2}{b_2^2}}] \tag{19}$$

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3.3 Maximum external reactance

In this section, we shall assume that we are given the mechanical input power (P_m) and the value of the capacitance (C) at the machine terminals. Then, the question we wish to answer is, "what is the maximum value of the external reactance (which we shall denote as x_e^{max}) to which the machine can deliver the specified power ?". This can be readily answered by rewriting the term Δ in Proposition 1 as a function of x_e. After a few algebraic manipulations, Δ can be arranged as a polynomial (6'th order) in x_e. Then the criterion for the maximum value of x_e is obtained from the following proposition.

Proposition 4 : Given the input power P_m and C, the maximum external reactance to which the machine can deliver power is given by

$$x_e^{max} = \eta \tag{20}$$

where η is the largest positive real root of the polynomial

$$f_3(x_e) = x_e^6\eta_1 + x_e^5\eta_2 + x_e^4\eta_3 + x_e^3\eta_4 + x_e^2\eta_5 + x_e\eta_6 + \eta_7 = 0 \tag{21}$$

The terms $\eta_i, i = 1, 2 \ldots 7$ are as described in the appendix.

Proof : The condition $\Delta > 0$ from Proposition 1 is equivalent to Eqn.(17) in proposition 3. Expressing the terms a_2, b_2 and a_3 in terms of x_e yields

$$\Delta = -b_2^4\{x_e^6\eta_1 + x_e^5\eta_2 + x_e^4\eta_3 + x_e^3\eta_4 + x_e^2\eta_5 + x_e\eta_6 + \eta_7\} \tag{22}$$

Note that if $f_3(x_e) < 0$, then $\Delta > 0$. The condition $f_3(x_e) < 0$ can be clearly met if $x_e < \eta$ which is the claim in this proposition.

Remarks :

1. In proposition 2, we assume P_m and x_e as specified (i.e. fixed) and calculate the minimum capacitance C_{min} as a function of P_m and x_e.
2. In proposition 3, we assume that C and x_e as specified and calculate the maximum power that P_m^{max} that the machine can deliver to the network.
3. In proposition 4, we assume that P_m and C as specified and calculate the maximum external reactance x_e to which the machine can deliver the specified power.

4. Note that all the three propositions directly follow from Proposition 1. The three propositions explicitly characterize the functional dependence of C, P_m and x_e for existence of equilibrium points when any two of them are held fixed. In the next section, we study the effect of capacitance on the power factor and terminal voltage regulation. And finally, we numerically study how the minimum required capacitance, maximum deliverable power and maximum reactance vary as a function of the their respective parameters.

4 Numerical Study

In this section, we conduct a numerical study of the analysis carried out in the previous section. In the study, three different sets of machine data are analysed. The machines correspond to squirrel cage induction generators used in wind turbines including one used in the Spanish system in the Galicia and Aragon-Navarra regions [1]. The relevant electrical machine data is provided in Appendix A. In the context of induction generators connected to the utility grids, constraints may be imposed on the units by regulatory agencies, or physical limitations of the turbine - generator unit. For example, the Spanish regulation ordains that the power factor at the interconnection bus be equal or greater than 0.86. The units may also have undervoltage trip protection (typically 0.85 p.u) and overspeed protection (typically 1.15 p.u) to protect them from potential damage. The purpose of the study is to obtain insight in to two prime factors that affect the performance of the unit which include (a) power factor and (b) voltage regulation. It is natural to expect the terminal voltage to depend on the capacitance of the compensator bank. In this study, a deviation in terminal voltage of $\pm 5\%$ from the nominal value of 1 p.u. is assumed as acceptable. Therefore, we denote C_{low} as the capacitance required to maintain the magnitude of the terminal voltage U at 0.95 p.u and C_{hi} as the corresponding value to maintain the terminal voltage at 1.05 p.u. The power factor variation is considered in the range $[0.85 - 1]$ keeping in mind the regulation mentioned earlier. However, for this study, we impose a tighter constraint on the variation in power factor and denote $[C'_{low}, C'_{hi}]$ as the capacitive range required to maintain the power factor above 0.95. For each of the cases considered in this study, a tabular summary of the capacitive range for voltage and power factor regulation is provided. Further, we shall study the impact of this capacitive range on (c) maximum transferable power, P_{max} using proposition 3 and (d) maximum external reactance to which the unit can deliver power using proposition 4. Along with this, the minimum capacitive requirements of the compensator bank for various combinations of line reactance and input power P_m is also studied using proposition 2. As we shall see later on, the study reveals the interaction of the four factors listed above and provides a means for understanding the limitations that arise due to the regulatory constraints. In what follows, we shall present the results for each of the cases considered.

4.1 Case I :

The machine data in this case corresponds to the induction generators used in the Spanish system [1]. The capacitance of the compensator bank is varied and the corresponding variations in terminal voltage and power factor are shown in Fig.3 and Fig.4 respectively. The maximum transferable power P_{max} and the maximum reactance to which the machine can deliver power are shown in Fig.5 and Fig.6 respectively. The minimum capacitance requirement is shown in Fig.7. The capacitive range for voltage and power factor regulation is summarised in Table 1.

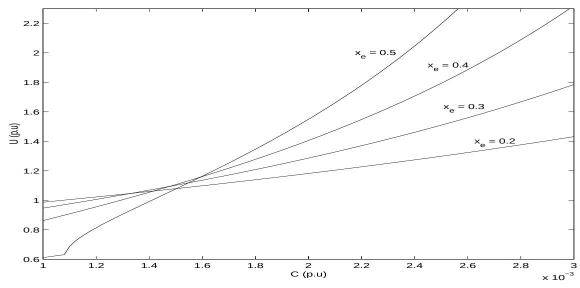

Figure 3: Terminal voltage variation with capacitance : Case I

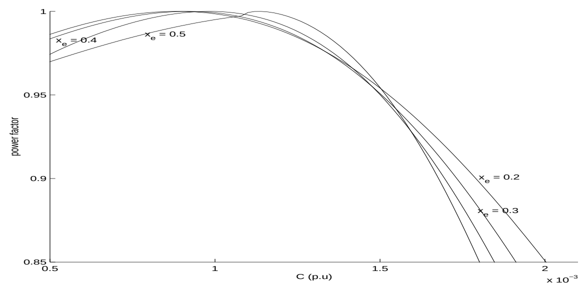

Figure 4: Power factor variation with capacitance : Case I

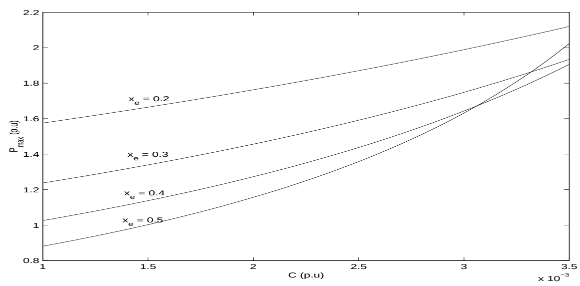

Figure 5: Maximum transferable power : Case I

4.2 Case II :

The machine data in this case corresponds to a small 350 kVA, 660 V induction generator used in wind turbine studies in [4]. The capacitance of the compensator bank is varied and the corresponding variations in terminal

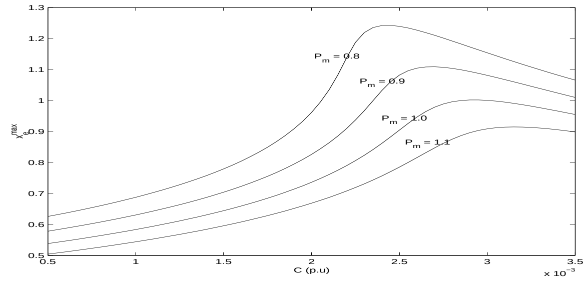

Figure 6: Maximum reactance to which machine can deliver power : Case I

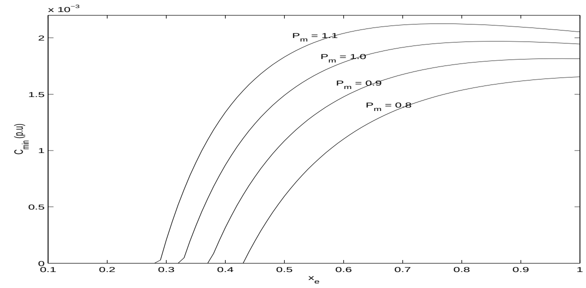

Figure 7: Minimum capacitance requirement : Case I

Table 1: Summary of capacitive range : Case I

line reactance	voltage regulation		p.f regulation	
	C_{low}	C_{hi}	C'_{low}	C'_{hi}
$x_e = 0.2$	0.76	1.34	0.1	1.53
$x_e = 0.3$	1.0	1.35	0.1	1.51
$x_e = 0.4$	1.2	1.39	0.3	1.50
$x_e = 0.5$	1.4	1.5	0.16	1.52

voltage and power factor are shown in Fig.8 and Fig.9 respectively. The maximum transferable power P_{max} and the maximum reactance to which the machine can deliver power are shown in Fig.10 and Fig.11 respectively. The minimum capacitance requirement is shown in Fig.12. The capacitive range for voltage and power factor regulation is summarised in Table 2.

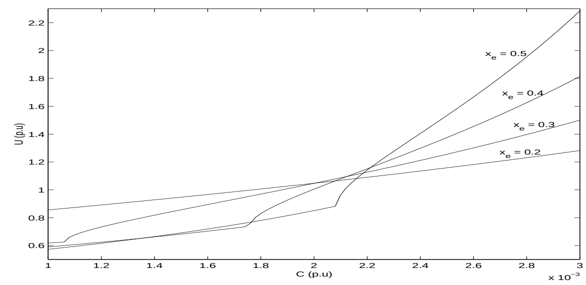

Figure 8: Terminal voltage variation with capacitance : Case II

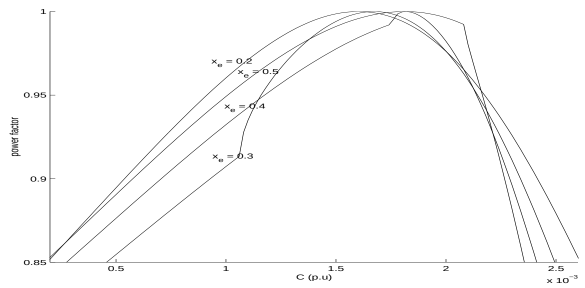

Figure 9: Power factor variation with capacitance : Case II

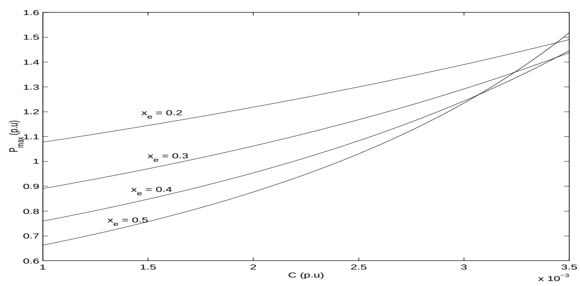

Figure 10: Maximum transferable power : Case II

4.3 Case III :

The machine data in this case corresponds to a large, 4 pole 6.6 kV induction generator used in the [11] and [12]. As the capacitance of the compensator bank is varied, the corresponding variations in terminal voltage

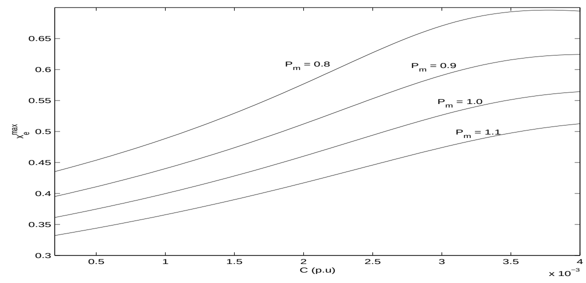

Figure 11: Maximum reactance to which machine can deliver power : Case II

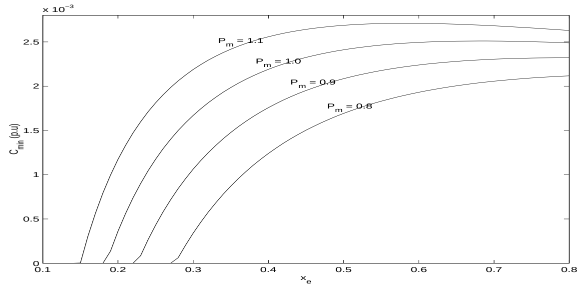

Figure 12: Minimum capacitance requirement : Case II

Table 2: Summary of capacitive range : Case II

line reactance	voltage regulation		p.f regulation	
	C_{low}	C_{hi}	C'_{low}	C'_{hi}
$x_e = 0.2$	1.5	2.0	0.92	2.11
$x_e = 0.3$	1.7	2.02	1.2	2.13
$x_e = 0.4$	1.9	2.09	1.2	2.15
$x_e = 0.5$	2.1	2.15	1	2.16

and power factor are shown in Fig.13 and Fig.14 respectively. The maximum transferable power P_{max} and the maximum reactance to which the machine can deliver power are shown in Fig.15 and Fig.16 respectively. The minimum capacitance requirement is shown in Fig.17. The capacitive range for voltage and power factor regulation is summarised in Table 3.

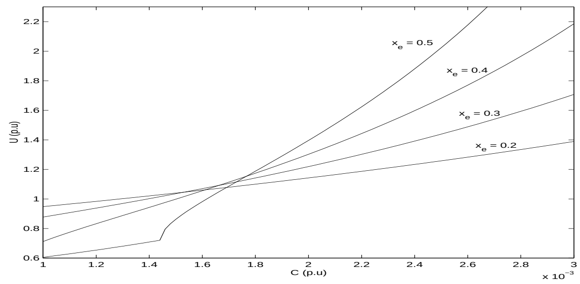

Figure 13: Terminal voltage variation with capacitance : Case III

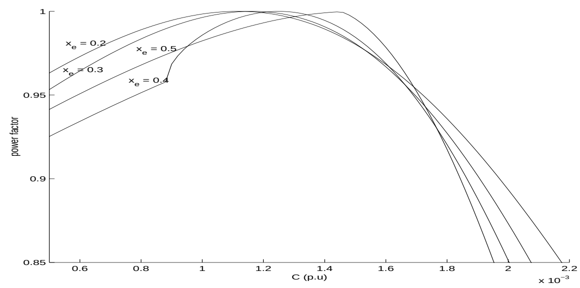

Figure 14: Power factor variation with capacitance : Case III

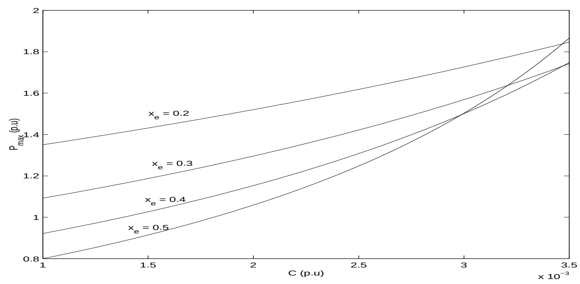

Figure 15: Maximum transferable power : Case III

5 Discussion

In this section, a discussion of the numerical results obtained in the previous section is provided.

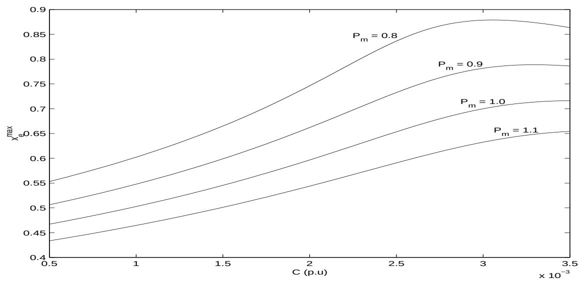

Figure 16: Maximum reactance to which machine can deliver power : Case III

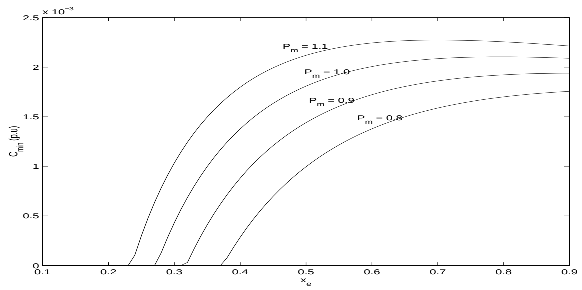

Figure 17: Minimum capacitance requirement : Case III

Table 3: Summary of capacitive range : Case III

line reactance	voltage regulation		p.f regulation	
	C_{low}	C_{hi}	C'_{low}	C'_{hi}
$x_e = 0.2$	1.0	1.59	0.38	1.64
$x_e = 0.3$	1.2	1.51	0.48	1.65
$x_e = 0.4$	1.4	1.62	0.76	1.69
$x_e = 0.5$	1.70	1.75	0.61	1.70

1. Terminal voltage variation with capacitance : Figs. 3, 8 and 13 depict the variation of terminal voltage with capacitance for the three cases considered. When $P_m = 0.9$, the capacitive limits C_{low}, C_{hi} (expressed in 10^{-3} p.u) for various values of the line impedance are summarised in Tables 1, 2 and 3. As one can note from these tables, the effective range of capacitance (=$C_{hi} - C_{low}$) shrinks quickly with increasing values of the external reactance. In the first case, when $x_e = 0.2$, we get a capacitive range of 0.64 $\times 10^{-3}$ p.u which reduces to 0.1×10^{-3} p.u when $x_e = 0.5$. In the second case, this range shrinks from 0.5 $\times 10^{-3}$ p.u to 0.05 $\times 10^{-3}$ p.u and in the third case, from 0.6 $\times 10^{-3}$ p.u to 0.05 $\times 10^{-3}$ p.u. In all three cases, we note that for values of x_e ranging from 0.2 to 0.5, the graphs intersect approximately at a point where the terminal voltage is slightly higher that 1.0 p.u. For example, in the first case considered, this point is noted to be approximately at C = 1.5 $\times 10^{-3}$ p.u. For the second and third cases, these points are C = 1.6 $\times 10^{-3}$ p.u and C = 2.1 $\times 10^{-3}$ p.u respectively. These observations indicate that the task of steady state terminal voltage regulation with changing external line reactance can be achieved with minimum capacitive switching effort if the capacitance of the compensator is set close to this value.

2. Power factor variation with capacitance : Figs. 4, 9 and 14 depict the variation of the power factor with capacitance for the three cases considered. In all three cases, we note that there is a fairly wide permissible range for the variation in capacitance of the compensator bank, keeping the power factor regulation in mind. Compare the range of capacitance for power factor regulation with that for voltage regulation as shown in Tables 1, 2 and 3 respectively. From the tables, it is clear that the power factor is in the agreeable range (i.e. 0.95) whenever the voltage is within $\pm 5\%$ of its nominal value (i.e. $0.95 < |U| < 1.05$). This suggests that power factor regulation is met whenever voltage regulation is met.

3. Maximum transferable power with capacitance : Figs. 5, 10 and 15 show how the maximum transferable power varies with capacitance for cases I, II and III respectively. For case I, we note from Table I that when $x_e = 0.2$, the maximum capacitance as dictated by voltage regulation is $C = 1.4 \times 10^{-3}$ p.u and for $x_e = 0.3, 0.4, 0.5$, this value is noted to be $C = 1.5 \times 10^{-3}$ p.u. Therefore, from Fig.5, we note that the maximum transferable power for $x_e = 0.2$ corresponds to that when $C = 1.4 \times 10^{-3}$ p.u which is close to 1.6 p.u. Similarly, for $x_e = 0.3, 0.4$ and 0.5, the maximum transferable powers are seen to be equal to 1.35, 1.14 and 1 respectively. A similar exercise can be repeated for cases II and III to obtain the maximum transferable powers for various values of the line reactance using the voltage/ power factor constrained values of compensator capacitance. Table 4 summarises the maximum power transfers for all the cases.

4. Maximum reactance with capacitance : Figs. 6, 11 and 16 show how the maximum reactance to which the machine can deliver a specified amount of power varies with the capacitance, for cases I, II and III respectively. In all three cases, we note that as the input power increases from 0.8 to 1.1, the maximum line reactance to which this power can be delivered reduces proportionately.

5. Minimum capacitive requirements : Figs.7, 12 and 17 depict how the minimum capacitance requirements (C_{min}) varies as a function of the line reactance x_e for different values of the input power P_m. Note that

Table 4: Summary of maximum power transfers

line reactance	case I	case II	case III
$x_e = 0.2$	1.62	1.2	1.45
$x_e = 0.3$	1.35	1.04	1.19
$x_e = 0.4$	1.14	0.96	1.06
$x_e = 0.5$	1.0	0.89	0.96

C_{min} is computed from Eq.(12). In the plot, C_{min} is set to zero whenever Eq.(12) yields a negative value of C_{min}. In all three cases, we note that at lower values of the line impedance x_e, capacitive compensation is not required for a feasible equilibrium. As the line reactance is increased for a fixed mechanical power input, capacitive compensation is required beyond a critical value of the line reactance. From the plots, it is consistently observed for all three cases that

(a) the critical line reactance reduces as the input power P_m is increased.

(b) the minimum capacitive requirements go up steeply at first, with increasing values of the line reactance and then saturate.

6 Conclusions

A system comprising of a wind energy converter (WEC) connected to the utility through a transmission line is studied. An induction generator employing a capacitive compensator bank is used to model the WEC. A third order dynamic model is used to represent the induction generator. The resultant system equations are analysed to derive a criterion which ensures the existence of an equilibrium or steady state solution to the system. This criterion is further analysed to compute (i) the minimum value of the capacitance of the compensator bank to deliver a specified value of real power to the given network, (ii) the maximum real power deliverable by the machine to the network and (iii) the maximum external reactance to which the machine can deliver power. This approach is seen to provide useful analytical insight in to the operation of an induction generator connected to an infinite bus. A numerical study of the effect of capacitance on terminal voltage regulation indicates that power factor regulation is achieved whenever terminal voltage regulation is achieved. It is seen that terminal voltage regulation has the strongest limiting influence on the power transfer and the weakest transmission line that can convey the power to the infinite bus. Thus, the analysis presented could be a useful tool for preliminary planning studies involving wind energy converters. Attempts are currently underway to build an experimental setup of an induction generator connected to the utility through a transmission line to obtain practical insight in to the operation of WECS and will be reported later on. The usefulness of FACTS compensation devices to simultaneously boost power transfers and achieve voltage/power factor regulation is another topic worth investigating.

Acknowledgements

The author wishes to thank the reviewers for their constructive comments and useful suggestions which have helped us enhance the quality of the manuscript. The author also thanks Prof. Andres Elias Feijoo for helpful discussions. The financial support from ND EPSCOR through NSF grant EPS 0132289 is gratefully acknowledged.

APPENDIX A : Induction generator data

Table 5: Electrical data for induction generators in p.u. (on machine base)

parameter	case I	case II	case III
x_r	0.143	0.0639	0.135
x_s	0.0087	0.1878	1.19
r_r	0.019	0.00612	0.0339
r_s	0.0059	0.00571	0.0059
x_M	4.76	2.78	4.161

The machine constants T_o', x' and x_o are defined as follows:
$T_o' = (x_r + x_m)/\omega_s r_r,\ x_o = x_r + x_m,\ x' = x_s + x_r x_m/(x_r + x_m)$.
The system parameters are assumed to be $E_b = 1.0$ p.u., $r_e = 0$.

APPENDIX B: Coefficients and parameters

In Eqn.(5), the parameters are described as follows.

$$a_{11} = r_e + r_s - x'Y_c r_e - Y_c r_s x_e$$

$$a_{12} = x_e + x' + Y_c r_s r_e - Y_c x' x_e$$

$$b_1' = E_r r_e - E_m x_e + E_b r_s,\ b_2' = E_m r_e + E_r x_e + E_b x'$$

In Eqns.(7), (8) and (9), the parameters are described as follows.

$$a_2 = x_e + x' - Y_c x_e x',\ \ b_1 = \frac{a_2 - x_e}{a_2 x'},\ \ x_a = x_o - x'$$

$$b_2 = \alpha_o(1 + x_a b_1)\ ,\alpha_o = \frac{1}{T_o'},\ \ a_3 = \frac{\alpha_o x_a E_b}{a_2},\ \ a_4 = \frac{E_b}{2Ha_2}$$

In proposition 3, the coefficients $\eta_1, \ldots \eta_7$ are defined as follows.

$$\eta_1 = K_1 c^4 b^2,\ \ \eta_2 = -2abc^4 K_1 + 4c^2 x' b^2 K_1$$

$$\eta_3 = K_1 a^2 c^4 + 4{x'}^2 b^2 K_1 + 2c^2 {x'}^2 b^2 K_1 - 8abc^2 x' K_1 - K_2 b^2 c^2$$

$$\eta_4 = 4c^2 a^2 x' K_1 + 4{x'}^3 b^2 K_1 - 4abc^2 {x'}^2 K_1 - 8{x'}^2 ab + 2abc^2 K_2$$

$$\eta_5 = 4{x'}^2 a^2 K_1 + 2c^2 a^2 {x'}^2 K_1 - 8ab{x'}^3 K_1 + {x'}^4 b^2 K_1 - K_2 a^2 c^2 - b^2 {x'}^2 K_2 + 4abcx' K_2$$

$$\eta_6 = -2ab{x'}^4 K_1 - 2cx' a^2 K_2 + 2ab{x'}^2 K_2$$

$$\eta_7 = K_1 {x'}^4 a^2 - K_2 {x'}^2 a^2$$

where,

$$a = \alpha_o(1 + x_a), \quad b = \alpha_o Y_c x_e, \quad c = 1 - Y_c x', \quad K_1 = 4P_m^2 \omega_s^2$$

$$K_2 = 4P_m \omega_s \alpha_o x_a E_b^2$$

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